Applied Mathematics Formula Sheet

Distance
1 foot = 12 inches
1 yard = 3 feet
1 mile = 5,280 feet
1 mile ≈ 1.61 kilometers
1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 meter = 1,000 millimeters
1 meter = 100 centimeters
1 kilometer = 1,000 meters
1 kilometer ≈ 0.62 miles

Area
1 square foot = 144 square inches
1 square yard = 9 square feet
1 acre = 43,560 square feet

Volume
1 cup = 8 fluid ounces
1 quart = 4 cups
1 gallon = 4 quarts
1 gallon = 231 cubic inches
1 liter ≈ 0.264 gallons
1 cubic foot = 1,728 cubic inches
1 cubic yard = 27 cubic feet
1 board foot = 1 inch by 12 inches by 12 inches

Weight
1 ounce ≈ 28.350 grams
1 pound = 16 ounces
1 pound ≈ 453.592 grams
1 milligram = 0.001 grams
1 kilogram = 1,000 grams
1 kilogram ≈ 2.2 pounds
1 ton = 2,000 pounds

Rectangle
perimeter = $2(\textit{length} + \textit{width})$
area = $\textit{length} \times \textit{width}$

Rectangular Solid (Box)
volume = $\textit{length} \times \textit{width} \times \textit{height}$

Cube
volume = $(\textit{length of side})^3$

Triangle
sum of angles = 180°
area = $\frac{1}{2}(\textit{base} \times \textit{height})$

Circle
number of degrees in a circle = 360°
circumference ≈ $3.14 \times \textit{diameter}$
area ≈ $3.14 \times (\textit{radius})^2$

Cylinder
volume ≈ $3.14 \times (\textit{radius})^2 \times \textit{height}$

Cone
volume ≈ $\dfrac{3.14 \times (\textit{radius})^2 \times \textit{height}}{3}$

Sphere (Ball)
volume ≈ $\frac{4}{3} \times 3.14 \times (\textit{radius})^3$

Electricity
1 kilowatt-hour = 1,000 watt-hours
amps = watts ÷ volts

Temperature
$°C = 0.56\,(°F - 32)$ or $\frac{5}{9}(°F - 32)$
$°F = 1.8\,(°C) + 32$ or $(\frac{9}{5} \times °C) + 32$

NOTE: Problems on the WorkKeys *Applied Mathematics* assessment should be worked using the formulas and conversions on this formula sheet.

ACT Math Facts & Formulas

Numbers, Sequences, Factors

Integers:	. . . , -3, -2, -1, 0, 1, 2, 3, . . .
Rationals:	fractions, that is, anything expressable as a ratio of integers
Reals:	integers plus rationals plus special numbers such as $\sqrt{2}$, $\sqrt{3}$ and π

Order Of Operations: PEMDAS
(Parentheses / Exponents / Multiply / Divide / Add / Subtract)

Arithmetic Sequences: each term is equal to the previous term plus d

Sequence: $t_1, t_1 + d, t_1 + 2d, \ldots$

Example: $d = 4$ and $t_1 = 3$ gives the sequence 3, 7, 11, 15, . . .

Geometric Sequences: each term is equal to the previous term times r

Sequence: $t_1, t_1 \cdot r, t_1 \cdot r^2, \ldots$

Example: $r = 2$ and $t_1 = 3$ gives the sequence 3, 6, 12, 24, . . .

Factors: the factors of a number divide into that number without a remainder

Example: the factors of 52 are 1, 2, 4, 13, 26, and 52

Multiples: the multiples of a number are divisible by that number without a remainder

Example: the positive multiples of 20 are 20, 40, 60, 80, . . .

Percents: use the following formula to find part, whole, or percent

$$\text{part} = \frac{\text{percent}}{100} \times \text{whole}$$

Example: 75% of 300 is what?
Solve $x = (75/100) \times 300$ to get 225

Example: 45 is what percent of 60?
Solve $45 = (x/100) \times 60$ to get 75%

Example: 30 is 20% of what?
Solve $30 = (20/100) \times x$ to get 150

ACT Math Facts & Formulas

Averages, Counting, Statistics, Probability

$$\text{average} = \frac{\text{sum of terms}}{\text{number of terms}}$$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{sum} = \text{average} \cdot (\text{number of terms})$$

$$\text{mode} = \text{value in the list that appears most often}$$

$$\text{median} = \text{middle value in the list}$$

median of $\{3, 9, 10, 27, 50\} = 10$
median of $\{3, 9, 10, 27\} = (9 + 10)/2 = 9.5$

Fundamental Counting Principle:

If an event can happen in N ways, and another, independent event can happen in M ways, then both events together can happen in $N \times M$ ways. (Extend this for three or more: $N_1 \times N_2 \times N_3 \ldots$)

Probability (Optional):

$$\text{probability} = \frac{\text{number of desired outcomes}}{\text{number of total outcomes}}$$

Example: each ACT math multiple choice question has five possible answers, one of which is the correct answer. If you guess the answer to a question completely at random, your probability of getting it right is $1/5 = 20\%$.

The probability of two different events A and B *both* happening is $P(A \text{ and } B) = P(A) \cdot P(B)$, as long as the events are independent (not mutually exclusive).

Powers, Exponents, Roots

$$x^a \cdot x^b = x^{a+b} \qquad x^a / x^b = x^{a-b} \qquad 1/x^b = x^{-b}$$

$$(x^a)^b = x^{a \cdot b} \qquad (xy)^a = x^a \cdot y^a \qquad (-1)^n = \begin{cases} +1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd.} \end{cases}$$

$$x^0 = 1 \qquad \sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

Factoring, Solving

$$(x+a)(x+b) = x^2 + (b+a)x + ab \qquad \text{"FOIL"}$$

$$a^2 - b^2 = (a+b)(a-b) \qquad \text{"Difference Of Squares"}$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$
$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

$$x^2 + (b+a)x + ab = (x+a)(x+b) \qquad \text{"Reverse FOIL"}$$

You can use Reverse FOIL to factor a polynomial by thinking about two numbers a and b which add to the number in front of the x, and which multiply to give the constant. For example, to factor $x^2 + 5x + 6$, the numbers add to 5 and multiply to 6, i.e., $a = 2$ and $b = 3$, so that $x^2 + 5x + 6 = (x+2)(x+3)$.

To solve a quadratic such as $x^2+bx+c = 0$, first factor the left side to get $(x+a)(x+b) = 0$, then set each part in parentheses equal to zero. E.g., $x^2 + 4x + 3 = (x+3)(x+1) = 0$ so that $x = -3$ or $x = -1$.

To solve two linear equations in x and y: use the first equation to substitute for a variable in the second. E.g., suppose $x + y = 3$ and $4x - y = 2$. The first equation gives $y = 3 - x$, so the second equation becomes $4x - (3 - x) = 2 \;\Rightarrow\; 5x - 3 = 2 \;\Rightarrow\; x = 1, y = 2$.

Solving two linear equations in x and y is geometrically the same as finding where two lines intersect. In the example above, the lines intersect at the point $(1, 2)$. Two parallel lines will have no solution, and two overlapping lines will have an infinite number of solutions.

Functions

A function is a rule to go from one number (x) to another number (y), usually written

$$y = f(x).$$

The set of possible values of x is called the *domain* of $f()$, and the corresponding set of possible values of y is called the *range* of $f()$. For any given value of x, there can only be one corresponding value y.

Absolute value:

$$|x| = \begin{cases} +x, & \text{if } x \geq 0; \\ -x, & \text{if } x < 0. \end{cases}$$

Logarithms (Optional):

Logarithms are basically the inverse functions of exponentials. The function $\log_b x$ answers the question: b to what power gives x? Here, b is called the logarithmic "base". So, if $y = \log_b x$, then the logarithm function gives the number y such that $b^y = x$. For example, $\log_3 \sqrt{27} = \log_3 \sqrt{3^3} = \log_3 3^{3/2} = 3/2 = 1.5$. Similarly, $\log_b b^n = n$.

A useful rule to know is: $\log_b xy = \log_b x + \log_b y$.

Complex Numbers

A complex number is of the form $a + bi$ where $i^2 = -1$. When multiplying complex numbers, treat i just like any other variable (letter), except remember to replace powers of i with -1 or 1 as follows (the pattern repeats after the first four):

$$\begin{array}{llll} i^0 = 1 & i^1 = i & i^2 = -1 & i^3 = -i \\ i^4 = 1 & i^5 = i & i^6 = -1 & i^7 = -i \end{array}$$

For example, using "FOIL" and $i^2 = -1$: $(1 + 3i)(5 - 2i) = 5 - 2i + 15i - 6i^2 = 11 + 13i$.

Lines (Linear Functions)

Consider the line that goes through points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Distance from A to B: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Mid-point of the segment $\overline{AB}$: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

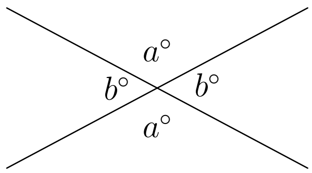
Slope of the line: $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{rise}}{\text{run}}$

Point-slope form: given the slope m and a point (x_1, y_1) on the line, the equation of the line is $(y - y_1) = m(x - x_1)$.

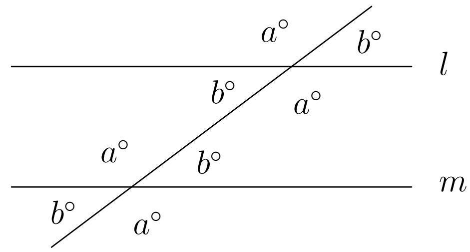
Slope-intercept form: given the slope m and the y-intercept b, then the equation of the line is $y = mx + b$.

To find the equation of the line given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, calculate the slope $m = (y_2 - y_1)/(x_2 - x_1)$ and use the point-slope form.

Parallel lines have equal slopes. Perpendicular lines (i.e., those that make a $90°$ angle where they intersect) have negative reciprocal slopes: $m_1 \cdot m_2 = -1$.



Intersecting Lines



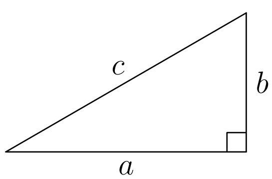
Parallel Lines ($l \parallel m$)

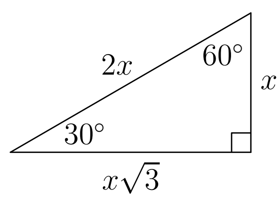
Intersecting lines: opposite angles are equal. Also, each pair of angles along the same line add to 180°. In the figure above, $a + b = 180°$.

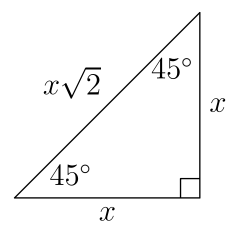
Parallel lines: eight angles are formed when a line crosses two parallel lines. The four big angles (a) are equal, and the four small angles (b) are equal.

Triangles

Right triangles:





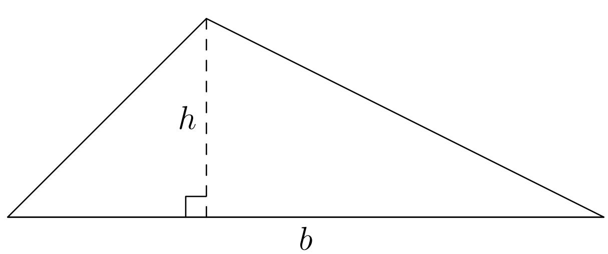


$a^2 + b^2 = c^2$

Special Right Triangles

A good example of a right triangle is one with $a = 3$, $b = 4$, and $c = 5$, also called a 3–4–5 right triangle. Note that multiples of these numbers are also right triangles. For example, if you multiply these numbers by 2, you get $a = 6$, $b = 8$, and $c = 10$ (6–8–10), which is also a right triangle.

All triangles:



$$\text{Area} = \frac{1}{2} \cdot b \cdot h$$

Angles on the inside of any triangle add up to 180°.

The length of one side of any triangle is always *less* than the sum and *more* than the difference of the lengths of the other two sides.

An exterior angle of any triangle is equal to the sum of the two remote interior angles.

Other important triangles:

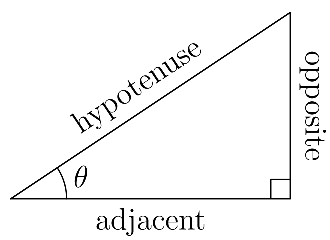
Equilateral: These triangles have three equal sides, and all three angles are 60°.

Isosceles: An isosceles triangle has two equal sides. The "base" angles (the ones opposite the two sides) are equal (see the 45° triangle above).

Similar: Two or more triangles are similar if they have the same shape. The corresponding angles are equal, and the corresponding sides are in proportion. For example, the 3–4–5 triangle and the 6–8–10 triangle from before are similar since their sides are in a ratio of 2 to 1.

Trigonometry

Referring to the figure below, there are three important functions which are defined for angles in a right triangle:



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

"SOH" "CAH" "TOA"

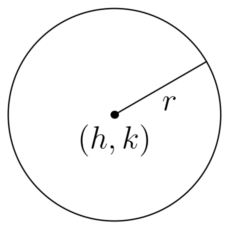
(the last line above shows a mnemonic to remember these functions: "SOH-CAH-TOA")

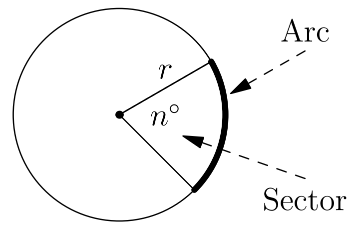
Optional: A useful relationship to remember which works for any angle θ is:

$$\sin^2\theta + \cos^2\theta = 1.$$

For example, if $\theta = 30°$, then (refer to the Special Right Triangles figure) we have $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$, so that $\sin^2 30° + \cos^2 30° = 1/4 + 3/4 = 1$.

Circles





$$\text{Area} = \pi r^2$$
$$\text{Circumference} = 2\pi r$$
$$\text{Full circle} = 360°$$

$$\text{Length Of Arc} = (n°/360°) \cdot 2\pi r$$
$$\text{Area Of Sector} = (n°/360°) \cdot \pi r^2$$

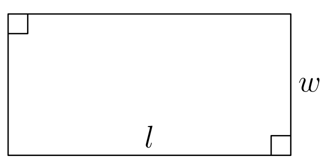
Equation of the circle (above left figure): $(x - h)^2 + (y - k)^2 = r^2$.

Another way to measure angles is with radians. These are defined such that π radians is equal to 180°, so that the number of radians in a circle is 2π (or 360°).

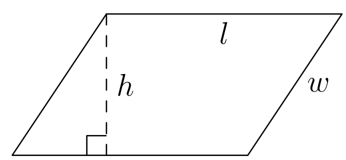
To convert from degrees to radians, just multiply by $\pi/180°$. For example, the number of radians in 45° is 0.785, since $45° \cdot \pi/180° = \pi/4 \text{ rad} \approx 0.785$ rad.

Rectangles And Friends

Rectangles and Parallelograms:

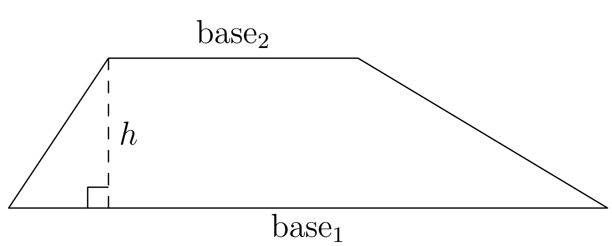


Rectangle
(Square if $l = w$)
Area $= lw$



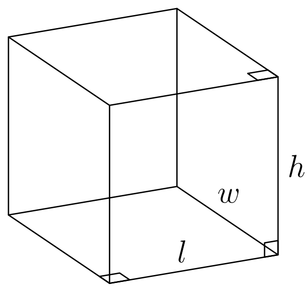
Parallelogram
(Rhombus if $l = w$)
Area $= lh$

Trapezoids (Optional):



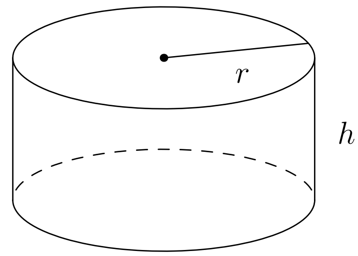
$$\text{Area of trapezoid} = \left(\frac{\text{base}_1 + \text{base}_2}{2}\right) \cdot h$$

Solids (Optional)



Rectangular Solid

Volume $= lwh$



Right Cylinder

Volume $= \pi r^2 h$

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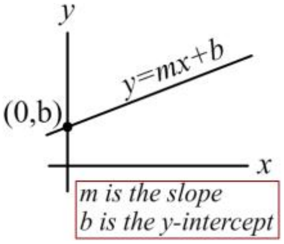
Expect NINE Coordinate Geometry problems on your ACT Math Test.
***Disclaimers**: Keep in mind that **this page does not cover every coordinate geometry concept** you may need on the ACT Math Test. Also, this page **<u>may contain errors</u>**. If you find an error, we'd love to hear about it: mathontime@gmail.com*

SLOPE of a LINE



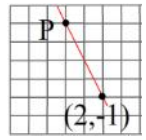
<u>Slope-intercept form</u> of a line: $y = mx + b$

Ex. Find the equation of a line with slope 7 that contains point (2,1). Use $m = 7$ in the equation above to get $y = 7x+b$. To find b, plug in the point (2,1) to get $1 = 7 \cdot 2 + b$ or $b = -13$. Thus $y = 7x-13$.

Ex. Put $6x + 3y = 1$ in slope-intercept form. Subtract $6x$ from both sides to get $3y = -6x+1$. Next, divide both sides by 3 to get the slope-intercept form $y = -2x+1/3$.

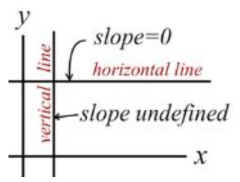
<u>Slope</u> of line containing (x_1, y_1) and (x_2, y_2): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Ex. A line through points (-1,4) and (2,5) has slope $m = \dfrac{5-4}{2-(-1)} = \dfrac{1}{3}$



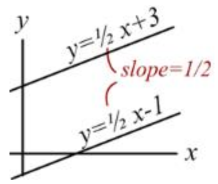
GRID Representation

Ex: Point (2,–1) is 4 units BELOW point P and 2 units to the RIGHT of P, so the slope of a line through these two points is m = rise/run = – 4/2 = – 2.



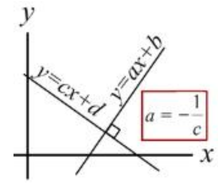
The slope of a <u>horizontal line</u> is 0. The slope of a <u>vertical line</u> is undefined.

Ex. y=5 is a horizontal line (zero slope), x= –3 is a vertical line (undefined slope).



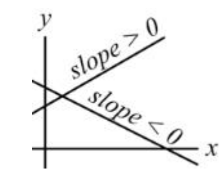
<u>Parallel lines</u> have the SAME slope.

Ex. The lines y= –10x–4 and y= –10x+200 are parallel since both have a slope equal to –10.



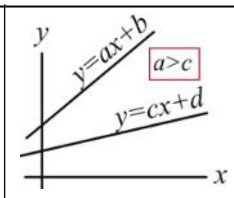
<u>Perpendicular lines</u> have slopes that are negative reciprocals.

Ex. y=2x–12 is perpendicular to y= –0.5x+3 since the neg. reciprocal of 2 is –1/2 = – 0.5.



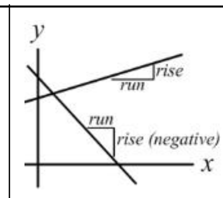
A line with <u>positive slope</u> goes up and to the right as x increases. A line with <u>negative slope</u> goes down as x increases.

Ex. y= –x+1 has slope –1 and goes down as x increases.



For two lines with <u>positive slopes</u>, the steeper line has larger slope (shown above).

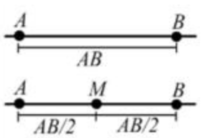
For two lines with <u>negative slopes</u>, the steeper line has a more negative slope.



SLOPE = Rise/Run
As x increases (<u>run</u> is positive), the <u>rise</u> will be either positive (for positive slope) or negative (for negative slope).

LINE SEGMENTS & MIDPOINT

- A line segment is defined by two points, A & B, on a line.
- A line segment has a specific length, *AB*.
- The <u>MIDPOINT</u>, *M*, of a line segment divides it in half.
- The midpoint between two points on the <u>real number line</u> is $(x_1 + x_2)/2$
- The midpoint between <u>two points in the x-y plane</u> is the point $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$



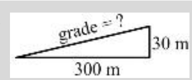
Ex. On the real number line, the midpoint between –3 and 12 is (12+(–3))/2 = 4.5

Ex. Find the point of <u>INTERSECTION</u> of the two LINES x-2y = 4 and 2x-3y = 9. Steps:
(1) Solve for x in first equation: $x = 4+2y$
(2) Plug this x into second equation: $2(4+2y)-3y = 9$
(3) Solve for y: $8+4y-3y=9$ or $y = 1$
(4) Plug $y=1$ back into 1st eq: $x - 2 \cdot 1 = 4$, so $x = 6$.
***Answer** (6,1) is the point where the lines intersect.*

The <u>GRADE</u> of a path or road is

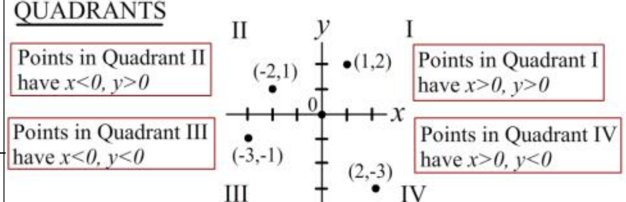
$$\text{Grade} = \frac{\text{rise}}{\text{run}} \cdot 100\%$$

Ex. Find the grade of a path that rises 30 m for every 300 m.



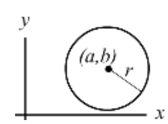
$$\frac{30}{300}(100\%) = 10\%$$

QUADRANTS



A <u>CIRCLE</u> with center at (a,b) and radius r has equation

$$(x-a)^2 + (y-b)^2 = r^2$$



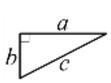
LENGTH & DISTANCE

The <u>distance</u> between points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is also the <u>length</u> of the line segment with these two endpoints.

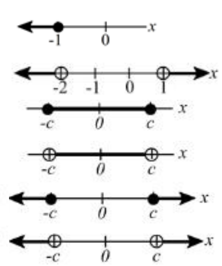
Another way to find <u>length</u> is to spot a <u>right triangle</u> with hypotenuse equal to the line segment of interest, and use the <u>Pythagorean Theorem</u> to find the length of the hypotenuse.

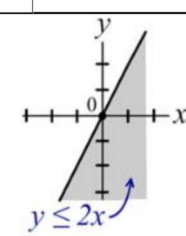


$$c = \sqrt{a^2 + b^2}$$

<u>NUMBER LINE GRAPHS</u>

Ex. $-1 \ge x$ (same as $x \le -1$)
Ex. $x < -2$ OR $x > 1$
$|x| \le c$ means $-c \le x \le c$
$|x| < c$ means $-c < x < c$
$|x| \ge c$ means $x \le -c$ or $x \ge c$
$|x| > c$ means $x < -c$ or $x > c$

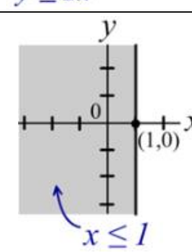




To show the region where $y \le 2x$, in the coordinate plane, graph the line

$$y = 2x$$

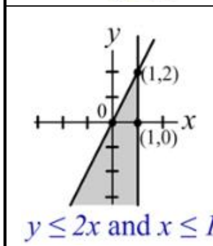
and shade the entire region BELOW this line.



To show $x \le 1$ graph the (vertical) line

$$x = 1$$

and shade to the LEFT of the line.

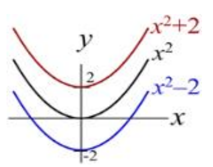


To show the system,

$$y \le 2x \text{ AND } x \le 1$$

graph BOTH lines and shade the region that is the INTERSECTION of the two individual regions. These points lie below y=2x <u>AND</u> left of x=1.

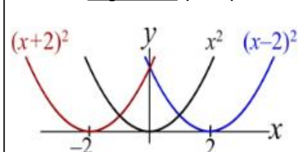
To shift a graph UP or DOWN, add or subtract a constant *c* (*c*>0):



To graph $f(x)+c$, shift the graph of $f(x)$ UP c units.

To graph $f(x)-c$, shift the graph of $f(x)$ DOWN c units.

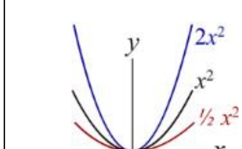
To shift a graph LEFT or RIGHT, subtract or add a constant to <u>argument</u> (*c*>0):



To graph $f(x\text{-}c)$, shift the graph of $f(x)$ to the RIGHT c units.

To graph $f(x+c)$, shift the graph of $f(x)$ to the LEFT c units.

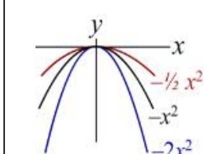
To STRETCH or COMPRESS a graph vertically (up and down), multiply by c:



If $c > 1$, the graph of $cf(x)$ looks like $f(x)$ <u>stretched</u> vertically (up & down) by a factor of c.

For $0 < c < 1$, $cf(x)$ looks like $f(x)$ <u>compressed</u> by c.

Multiplying by c<0, FLIPS a graph upside down:



For $c < 0$, the same rules apply, plus the graph is flipped upside down (what was positive becomes negative and vice versa).

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TRIANGLES

For ALL TRIANGLES, the 3 angles add to 180° and Area A=(1/2) *base X height*

Equilateral Triangle has 3 identical sides and 3 identical angles of 60° (since 60°+60°+60°=180°).

A Right Triangle has one 90° angle, sides that satisfy $a^2+b^2=c^2$ (Pythag. Thm.), & the other 2 angles are <90°

Isosceles Triangle has 2 identical sides & 2 identical angles, as shown.

45°–45°–90° Right Triangle has sides with ratio $1:1:\sqrt{2}$

30°–60°–90° Right Triangle has sides with ratio $1:2:\sqrt{3}$ (hypotenuse = 2)

3 – 4 – 5 Right Triangle has sides proportional to 3, 4 & 5.

Congruent Triangles have identical side lengths and angles. They may be rotated or reflected relative to one another and still be congruent.

Similar Triangles have identical corresponding angles, $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$ & proportional side lengths, $a/x = b/y = c/z$

Ex. Find x in degrees.

The 3 angles add to 180°, so

$x + 50° + 30° = 180°$

$x = 180° - 50° - 30° = 100°$

Ex. Find h for the right triangle shown.

The angles must add to 180° so this is a 45-45-90 right triangle. Thus the sides are in the ratio $1:1:\sqrt{2}$, or $7:7:7\sqrt{2}$, so $h = 7\sqrt{2}$.

Ex. A rectangle has sides of length 3 and 2 inches. What is the diameter of the rectangle?

The diameter divides the rectangle into two right triangles with legs 2 & 3, so

$d = \sqrt{3^2 + 2^2} = \sqrt{13} \approx 3.6$ in.

Ex. The two triangles shown are similar triangles. What is the perimeter of the larger triangle?

The side of length 9 corresponds to the side of length 3, so the side of length 6 and the unknown side must correspond to the two sides labeled 2. Thus the unknown side is 6 and *P* = 6+6+9 = 21.

Ex. If a right triangle has hypotenuse 15 mm and one leg 12 mm, find the length of the other leg.

Pythagorean thm. says $12^2 + x^2 = 15^2$

So $x = \sqrt{225 - 144} = 9$ mm.

ANGLES

Right Angles measure 90°

Right angles are indicated by a little square.

Supplementary angles add to 180°

Ex. $x + 148° = 180°$, so $x = 32°$

Vertical angles are congruent

Ex. θ = 85°

For parallel lines cut by a transversal, alternate interior angles are congruent. **Ex.** θ = 50°

Ex. Given AC is parallel to DF, AE is congruent to EB, and ∠CBE is 130°, as shown, find ∠AEF.

$\angle ABE = 50°$ since it is supplementary to 130°.

ΔABE is isosceles since *AE* is congruent to *BE*. $\angle BAE = 50°$ since base angles of isosceles triangle are congruent. *AE* is a transversal between parallel lines, so alternate interior angles are congruent. Thus $\angle AEF = 50°$

CIRCLES

Area *A* = π *r*², Circumference = 2 π *r*, diam. *d* = 2 *r*

The perpendicular bisector of a chord in a circle passes through the center of the circle.

Measure of arc $AB = x°$

Length of arc $AB = 2\pi r(x/360°)$

Ex. A circle has diameter 5 cm. Find the area.

$A = \pi r^2$ and $d = 2r$, so $A = \pi (5/2)^2 = 25\,\pi/4$ cm².

Ex. A chord in a circle has length 8" and is 3" from the center of the circle. Find the radius of the circle.

The line segment labeled 3" is perpendicular to the chord, so it bisects it. A right triangle is formed with the hypotenuse equal to the radius. This is a 3-4-5 right triangle, so *r* = 5 in.

Ex. A circle has diameter d=8 ft and θ =120°. Find the length of arc RS.

The radius is *r* = *d* / 2 = 4 ft, so

$\widehat{RS} = 2\pi r \dfrac{\theta}{360°} = 2\pi(4)\cdot\dfrac{120°}{360°} = \dfrac{8}{3}\pi \approx 8.4$ ft.

POLYGONS

Interior angles of n-sided polygon add up to $(n-2)\,180°$.

Perimeter of a polygon is the sum of the side lengths.

A Regular polygon has equal side lengths and equal angles each measuring $(n-2)\,180°/n$

A Quadrilateral is a 4-sided polygon. The angles of a quadrilateral sum to $(4-2)\,180° = 360°$

A Trapezoid is a quadrilateral with one pair of parallel sides. Find area by dividing into simpler shapes.

A Parallelogram is a quadrilateral with two pairs of parallel sides. Opposite sides are equal, opposite angles have equal measure.

A Regular Pentagon is a 5-sided polygon with equal sides and angles. The five angles each measure (5-2)180°/5=108°.

Square with side length s has Area = s², Perimeter = 4s

A Rectangle has Area = *l*·*w*, Perimeter = 2*l*+2*w*

Ex. Find the perimeter of the polygon given that all sides meet at 90°.

The unlabeled lengths are found by making sure the total length on one side of the figure adds up to the total length on the opposite side, (2+**2**=4 and 2+**1**=3).

Perimeter = 3+4+2+2+1+2 = 14

Ex. Three of the (interior) angles of a quadrilateral are 75°, 45°, and 125°. Find the fourth interior angle.

In a quadrilateral, the four angles must add to 360°, so the 4th angle is 360°- 75°- 45°- 125° = 115°

Ex. A parallelogram has one interior angle of 100°. Find the other 3 interior angles.

For a parallelogram, the 4 angles add to 360° and opposite angles have the same measure. Thus the angle opposite the 100° angle is also 100°, and the other two are both θ where $\theta + \theta + 100° + 100° = 360°$, or $\theta = 160°/2 = 80°$. Answer: 100°, 80°, 80°

TRANSLATION, ROTATIONS, REFLECTIONS

A translation slides an object to another location (no rotation).

A reflection flips an object across a line (often x or y-axis) to create a mirror image.

A rotation rotates an object about a point (often the origin).

Ex. This triangle is reflected across the x-axis to form a new triangle. Find the three corner points of the new triangle.

The x-coordinates of all three points remain the same, while the y-coordinate changes sign. (think about Quadrants.) Thus the point (3,1) becomes (3, - 1) when reflected, and the other two are (-1, - 1) and (2, - 2).

VOLUME AND AREA EXAMPLES

Ex. A shape is drawn on a grid with unit squares. Find the area in square units.

Divide the area into simpler shapes & count units along edges of each shape for dimensions:

Area = **triangle** + **square** + **rectangle**

$= 1/2\ (2\times3) + (3\times3) + (5\times1)$

$= 17$ square units

Ex. If one square tile is needed to cover an 8"X8" area, how many are needed to cover a 10' by 20' floor?

Convert floor dimensions to inches by multiplying by 12 to get 120"X240". Along the 120" side, 120/8=15 tiles are needed, and along the 240" side, 240/8=30 tiles are needed. So covering the rectangular floor requires a total of 15 X 30 or 450 tiles.

Ex. A circle is inscribed inside a square. Find the shaded area if the circle has area 9π m².

Since the area of the circle is πr²=9π, the radius of the circle is r=3 m. The side length of the square is the diameter of the circle, s=2r=6 m. Thus the square has area 6X6 = 36 m². The shaded area is the area of the square minus the area of the circle, 36 – 9π m².

Ex. Two identical cubes with 2 ft edges are stacked. Find the surface area and volume of the resulting figure.

The front, back, and two sides are identical with surface areas 4'X2'. The top and bottom have surface area 2'X2'. Total surface area is the sum of the 6 sides,

$4(4'\times2') + 2(2'\times2') = 32 + 8 = 40$ ft²

The volume is $V = l\cdot w\cdot h = 4\cdot2\cdot2 = 16$ ft³

Ex. A box-shaped aquarium with a height of 20 inches holds 6000 cubic inches of water when filled to the top. How much water does it hold when filled to a height of just 12 inches?

The volume of water held by the aquarium is $V = l\cdot w\cdot h$, but the length and width are the same whether the tank is filled to 20 or 12 inches. Thus the following proportion will give the desired volume:

12 is to 20 as V is to 6000 or

$12/20 = V/6000$

Solving gives $V = 12(6000)/20 = 3600$ in³